# Hadron spin-flip amplitude: an analysis of the new $A_N$ data from RHIC

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Abstract

Through a direct analysis of the scattering amplitude, we show that the preliminary measurement of $A_N$ obtained by the E950 Collaboration at different energies are mainly sensitive to the spin-flip part of the amplitude in which the proton scatters with the $^{12}C$ nucleus as a whole. The imaginary part of this amplitude is negative, and the real part positive. We give predictions for $p_L = 600$ GeV$/c$, which depend mainly on the size of the real part of the amplitude.

Diffractive polarized experiments open a new window on the spin properties of QCD at large distances. In particular, the recent data from RHIC and HERA indicate that, even at high energy, the hadronic amplitude has a significant spin-flip contribution, $\mathcal{A}^h_{sf}$, which remains proportional to the spin-non-flip part, $\mathcal{A}^h_{nf}$, as energy is increased [1, 2]. In other words, the pomeron coupling to the proton and/or to the nuclei has a non-trivial spin structure.

The new RHIC fixed-target data, from E950, consists in measurements of the analysing power

$$A_N(t) = \frac{\sigma^{(\uparrow)} - \sigma^{(\downarrow)}}{\sigma^{(\uparrow)} + \sigma^{(\downarrow)}} \tag{1}$$

for momentum transfer $0 \leq |t| \leq 0.05$ GeV$^2$, for a polarized $p$ beam hitting a (spin-0) $^{12}C$. In this region of $t$, the electromagnetic amplitude is of the same order of magnitude as the hadronic amplitude, and the interference of the imaginary part of $\mathcal{A}^h_{nf}$with the spin-flip part of the electromagnetic

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amplitude $\mathcal{A}_{sf}^{em}$leads to a peak in the analyzing power $A_N$, usually referred to as the Coulomb-Nuclear Interference (CNI) effect [3, 4, 5]. This effect was observed in the data from [6], but the errors were too big to draw any conclusion on the hadron spin-flip amplitude.

The first RHIC measurements at $p_L = 22$ GeV/$c$ [7] in $p^{12}C$ scattering indicated however that $A_N$ changes sign already at very small momentum transfer. Such a behaviour cannot be described by the CNI effect alone. Indeed, fits to the data [8] give for

$$\begin{aligned} r_5 &= \lim_{t\to 0} \tilde{\mathcal{A}}^h{}_{sf}/Im(\mathcal{A}^h_{nf}) \equiv R + iI : &(2)\\ R &= 0.088 \pm 0.058; \quad I = -0.161 \pm 0.226 &(3) \end{aligned}$$

As usual, $\tilde{\mathcal{A}}_{sf}(s,t) \equiv 2\ m_p\ \mathcal{A}_{sf}(s,t)/\sqrt{|t|}$ is the "reduced" spin-flip amplitude factoring out trivial kinematical factors. The large error on $Im(r_5)$ unfortunately leads to a high uncertainty on the size of the hadronic spin-flip amplitude.

Before analysing the the new (preliminary) data at $p_L = 24$ and 100 GeV/$c$ [9], we need to define the ingredients of the theoretical description. Isoscalar targets such as $^{12}C$ simplify the calculation as they suppress the contribution of the isovector reggeons $\rho$ and $a_2$, by some power of the atomic number. Also, as $^{12}C$ is spin 0, there are only two independent helicity amplitudes: proton spin flip and proton spin non flip. However, nuclear targets lead to large theoretical uncertainties because of the difficulties linked to nuclear structure, and because of the lack of high-energy proton-nucleus scattering experiments (see, for example, [10]). Given these problems, we shall not rely on theoretical models (such as the Glauber formalism) but rather parametrise the scattering amplitude directly from data, and take the interference terms fully into account.

The elastic and total cross sections and the analysing power $A_N$ for $p^{12}C$ scattering are given by

$$\begin{aligned} d\sigma/dt &= 2\pi\left(|\mathcal{A}_{nf}|^2 + |\mathcal{A}_{sf}|^2\right), \\ \sigma_{tot} &= 4\pi Im(\mathcal{A}_{nf}), \qquad (4)\\ A_N\ d\sigma/dt &= -2\pi Im[\mathcal{A}_{nf}\mathcal{A}^*_{sf})]. \end{aligned}$$

Each term includes a hadronic and an electromagnetic contribution: $\mathcal{A}_i(s,t) = \mathcal{A}_i^h(s,t) + \mathcal{A}_i^{em}(t)e^{i\delta}$, $(i = nf, sf)$, where $\mathcal{A}_i^h(s,t)$ describes the strong interaction of $p^{12}C$, and $\mathcal{A}_i^{em}(t)$ the electromagnetic interaction. $\alpha_{em}$ is the electromagnetic fine structure constant, and the Coulomb-hadron phase $\delta$ is given by $\delta = Z\alpha_{em}\varphi_{CN}$ with $Z$ the charge of the nucleus, and $\varphi_{CN}$ the Coulomb-nuclear phase [11]. The electromagnetic part of the scattering amplitude can be written as

$$\begin{aligned} \mathcal{A}^{em}_{nf} &= \frac{2\alpha_{em}\, Z}{t}\, F^{^{12}C}_{em} F^{p}_{em1}, \\ \mathcal{A}^{em}_{sf} &= -\frac{\alpha_{em}\, Z}{m_p\sqrt{|t|}}\, F^{^{12}C}_{em} F^{p}_{em2}, \end{aligned} \tag{5}$$

where $F^p_{em1}$ and $F^p_{em2}$ are the electromagnetic form factors of the proton, and $F^{^{12}C}_{em}$ that of $^{12}C$. We use

$$F^p_{em1} = \frac{4m_p^2 - t(\kappa_p + 1)}{(4m_p^2 - t)(1 - t/0.71)^2}, \tag{6}$$

$$F^p_{em2} = \frac{4m_p^2 \kappa_p}{(4m_p^2 - t)(1 - t/0.71)^2}, \tag{7}$$

where $m_p$ is the mass of the proton and $\kappa_p$ its anomalous magnetic moment. We obtain $F^{^{12}C}_{em}$ from the electromagnetic density of the nucleus

$$D(r) = D_0 \left[1 + \tilde{\alpha}\left(\frac{r}{a}\right)^2\right] e^{-\left(\frac{r}{a}\right)^2}. \tag{8}$$

$\tilde{\alpha} = 1.07$ and $a = 1.7$ fm give the best description of the data [12] in the small-$|t|$ region, and produce a zero of $F^{^{12}C}_{em}$ at $|t| = 0.130$ GeV$^2$. We also calculated $F^{^{12}C}_{em}$ by integration of the nuclear form factor given by a sum of Gaussians [13] and obtained practically the same result with the zero now at $|t| = 0.133$ GeV$^2$.

The parts of the scattering amplitudes due to strong interaction are assumed to be well approximated by falling exponentials in the small-$t$ region. The slope parameter $B(s,t)/2$ is then the derivative of the logarithm of the amplitude with respect to $t$. If one considers only one contribution to the amplitude, this coincides with the slope of the differential cross section. In a more complicated case, there is no direct correspondance with the cross section because of interference terms.

The hadron spin-non flip amplitude consists of two parts, describing respectively the scattering of the proton on separate nucleons in the nucleus, and the scattering of the proton on the nucleus as a whole:

$$\mathcal{A}^h_{nf}(s,t) = \mathcal{A}^{pN}_{nf}(s,t) + \mathcal{A}^{pA}_{nf}(s,t), \tag{9}$$

$$
\begin{aligned}
\mathcal{A}^{pN}_{nf}(s,t) &= (1+\rho^{pN})\frac{\sigma^{pN}_{tot}(s)}{4\pi}\exp\left(\frac{B^{pN}}{2}t\right), \qquad (10)\\
\mathcal{A}^{pA}_{nf}(s,t) &= (1+\rho^{pA})\frac{\sigma^{pA}_{tot}(s)}{4\pi}\exp\left(\frac{B^{pA}}{2}t\right). \qquad (11)
\end{aligned}
$$

The size and energy dependence of $\rho^{pN}$ and $B^{pN}$ are assumed to be the same as in the $pp$ case [14]:

$$
\begin{aligned}
\rho^{pN}(s) &= 6.8/p_L^{0.742} - 6.6/p_L^{0.599} + 0.124, \qquad (12)\\
B^{pN}(s) &= 11.13 - 6.21/\sqrt{p_L} - 0.3\ \ln p_L.
\end{aligned}
$$

For $\sigma^{pN}_{tot}(s)$, we use the fact that $\sigma^{pN}_{tot} \approx 2\sigma^{pp}_{tot}$ and take the best form obtained in [15], which works well in this energy region:

$$
\begin{aligned}
\sigma^{pN}_{tot}(s) &\approx 2\ \sigma^{pp}_{tot}(s) \qquad (13)\\
&= 86\ (s/s_1)^{-0.46}\ - 66\ (s/s_1)^{-0.545}\\
&\quad +71 + 0.614\ \ln^2(s/s_0), \qquad (14)
\end{aligned}
$$

with all coefficients in mb, $s_1 = 1$ GeV$^2$ and $s_0 = 29$ GeV$^2$. The factor 2 in (13) reflects a complicated hadron-hadron interaction in the nucleus and is determined from the comparison of our calculation of $d\sigma/dt$ with the data of $p^{12}C$ scattering [16] in the region $0.1 \leq |t| \leq 0.2$ GeV$^2$, where the term $\mathcal{A}^{pN}_{nf}(s,t)$ of the scattering amplitude gives the main contribution.

For the determination of $\mathcal{A}^{pA}_{nf}(s,t)$, we rely on the data obtained by the SELEX Collaboration. This experiment on $pC$ scattering at $p_L = 600$ GeV/$c$ gives us $\sigma^{pC}_{tot}\ =\ 341$ mb; $B^{pC}(t \approx 0.02\ \mathrm{GeV}^2) = 62\ \mathrm{GeV}^{-2}$.

To obtain the values of these parameters for other energies, we make the following assumptions on their energy dependence: some analyses [17] and the data [18] show that the ratio $R_{C/p}$ of $\sigma_{tot}(p^{12}C)$ to $\sigma_{tot}(pp)$ decreases very slowly in the region $5 \leq p_L \leq 600$ GeV/$c$. We take its energy dependence, according to the data [16], as $R_{C/p} = 9.5\ (1 - 0.015 \ln s)$. From this we obtain $\sigma^{pA}_{tot}(s) \equiv \sigma^{pC}_{tot}(s) - \sigma^{pN}_{tot}(s)$. We assume that the slope slowly rises with $\ln s$ in a way similar to the $pp$ case, and normalise it so that the full amplitude (9) has a slope of 62 GeV$^{-2}$ at $p_L = 600$ GeV/c and $|t| = 0.02$ GeV$^2$. This gives $B^{pA} = 70\ (1 + 0.05\ \ln s)$.

We do not know the energy dependence of $\rho^{pA}$, but because the $\rho$ and $a_2$ trajectories are suppressed, and because they contribute negatively, it must be larger than in the $pp$ case, where it is about $-0.1$ in this energy region. In fact, the data from [14, 16] indicate that $\rho^{pA}$ is positive. We also know that at very high energy, $\rho^{pA}$ should be of the order of $\rho^{pp}$, which is about 0.1. We thus assume that at RHIC energies, it is of the order of 0.05, and that it changes logarithmically with $s$, similarly to the $pp$ case. We also allow for an extra term proportional to $\rho_{pp}$ with a linear suppression in $A$. This gives us two variants:

$$\rho^{pA} = 0.05/(1-0.05\ \ln s) + \rho_{pp}/A, \tag{15}$$

$$\rho^{pA} = 0.05/(1-0.05\ \ln s). \tag{16}$$

At small transfer momenta $|t| \leq 0.03$ GeV$^2$, the ratio of the effective hadronic form factor $\mathcal{A}^h_{nf}(s,t)/\mathcal{A}^h_{nf}(s,0)$ to the effective electromagnetic form factor $\mathcal{A}^{em}_{nf}(s,t)/\mathcal{A}^{em}_{nf}(s,0)$ is roughly equal to 1 and grows slowly to 1.25 at $|t| = 0.05$ GeV$^2$.

The hadron spin-flip amplitude of $p^{12}C$ scattering is mainly due to the interaction of the proton with the nucleus as a whole. The spin-dependent scattering of the proton on one separate nucleon of $^{12}C$ is averaged to zero by the nuclear wave function. Scattering on multiple nucleons is suppressed because these must resum to spin zero and because the anomalous magnetic moments of the proton and of the neutron have opposite signs. We show in Figs. 2 and 3 the (small) additional effect of a $pN$ contribution set to 10% of the $pp$ spin non flip:

$$\tilde{\mathcal{A}}^{pN}_{sf}(s,t) = \mathcal{A}^{pN}_{nf}(s,t)/10. \tag{17}$$

We parametrise the remainder spin-flip part of $p^{12}C$ scattering as

$$\begin{aligned} \mathcal{A}^h_{sf}(s,t) &= (k_2\rho^{pA} + ik_1)\frac{\sqrt{|t|}\sigma^{pA}_{tot}(s)}{4\pi} \\ &\quad \times \exp\left(\frac{B^{pA}}{2}t\right). \end{aligned} \tag{18}$$

We have assumed here that the spin-flip and the spin-non-flip amplitude have the same slope. One could of course allow for more freedom and take different slopes, but the data are not yet precise enough to test this.

From the full scattering amplitude, the analyzing power is given by

$$A_N\frac{d\sigma}{dt} = -4\pi[Im(\mathcal{A}_{nf})Re(\mathcal{A}_{sf}) - Re(\mathcal{A}_{nf})Im(\mathcal{A}_{sf})],$$

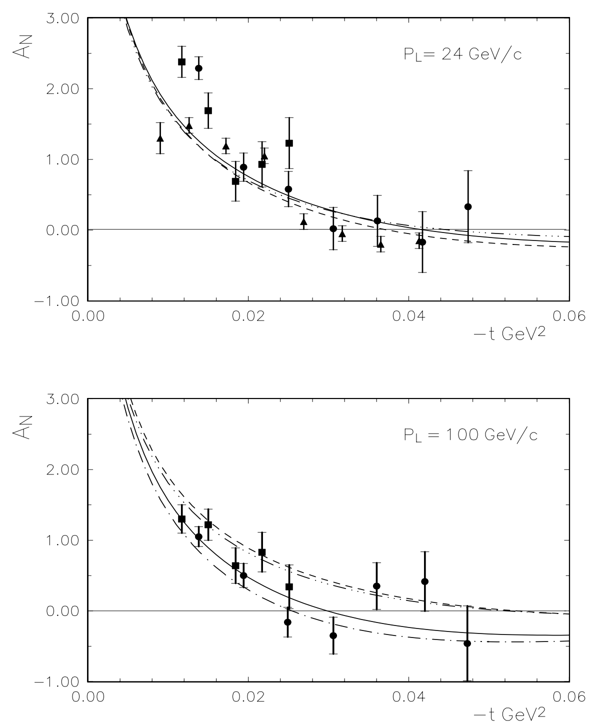


Figure 1: The analysing power $A_N$ (in %) at $p_L = 24$ GeV/$c$ and 100 GeV/$c$, compared with the data [7, 9] (only statistical errors are shown). The two scenarions (15) and 16 lead respectively to the upper and lower curves (these are indistinguishable at 24 GeV/$c$). The dot-dashed curves correspond to the addition of a small spin-flip contribution from nucleons (17).

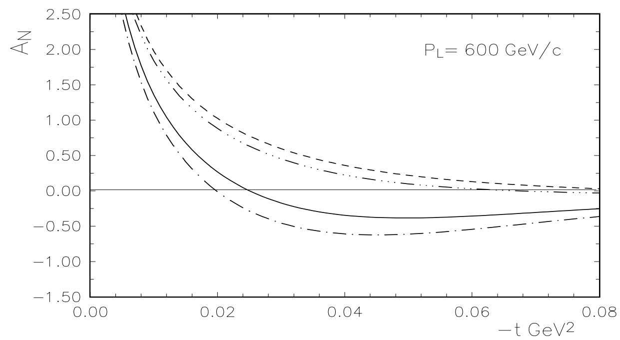


Figure 2: The predictions for $A_N$ (in %) at $p_L = 600$ GeV/$c$. The curves are as in Fig. 1.

each term having electromagnetic and hadronic contributions. We can now calculate the form of the analyzing power $A_N$ at small momentum transfer with different coefficients $k_1$ and $k_2$ chosen to obtain the best description of $A_N$ at $p_L = 24$ GeV/$c$ and $p_L = 100$ GeV/$c$. Of course, we only aim at a qualitative description as the data are only preliminary and as they are normalized to those at $p_L = 22$ GeV/$c$ [7].

The preliminary data show that $A_N$ decreases very fast after its maximum and is almost zero in a large region of momentum transfer. This behaviour can be explained only if one assumes a negative contribution of the interference between different parts of the hadron amplitude, that changes slowly with energy.

The data at $p_L = 100$ GeV/$c$ decrease faster than those at $p_L = 24$ GeV/$c$, and the zero of $A_N$ moves to lower values of $|t|$. This change of sign is independent from the normalization of the data. It would be very interesting to obtain new data with higher accuracy and at higher energies in order to distinguish between the two scenarios (15) and (16).

The best descriptions of the data, shown in Fig. 1, lead to the values of $R$ and $I$ given in Table I.

The two sets of parameters $k_1$ and $k_2$, chosen to obtain equivalent descriptions of the data at $p_L = 24$ GeV/c (Fig. 1), produce quite different curves at higher energies, as can be seen in Figs. 1 and 2, for $p_L = 100$ GeV/$c$ and 600 GeV/$c$. The difference between the two variants with different energy dependence of $\rho$ grows and reaches 1% at $|t| = 0.03$ GeV$^2$ and $p_L = 600$ GeV/c.

| $p_L$ (GeV) | form of $\rho$ | I | R |
|---|---|---|---|
| 24 | (15) | -0.146 | 0.098 |
| | (16) | -0.146 | 0.092 |
| 100 | (15) | -0.145 | 0.137 |
| | (16) | -0.145 | 0.086 |
| 600 | (15) | -0.144 | 0.162 |
| | (16) | -0.143 | 0.080 |

Table 1: Energy dependence of $I$ and $R$ from our calculations.

Around the maximum of the Coulomb-hadron interference, this difference is very small, but it grows for $|t| > 0.02$ GeV$^2$. More importantly, the two variants lead to values of $A_N$ with opposite signs. This characteristic is obviously independent on the normalisation of the data. So, a determination of the size and energy dependence of the spin-flip amplitude requires a good knowledge of $\rho^{pA}$.

Both variants give the same size and negative sign for the imaginary part of the spin-flip amplitude, as shown in Fig. 3. As mentioned above, such an amplitude gives an additional positive contribution to the CNI-effect at the maximum. Its size is mostly determined by the magnitude of $A_N$ at small $|t|$. The fast change of sign of $A_N$ is explained by the interference of different parts of the hadronic amplitude.

Hence, the shape and energy dependence of the analyzing power depend mostly on the size and energy dependence of $\rho_{pC}$. If we choose another size and energy dependence, we can obtain a different shape for $A_N$ and different magnitudes for $k_1$ and $k_2$. However all conclusions will stand and $I = Im(r_5)$ will remain negative. Note that a positive $Im(r_5)$ would lead to an increase with energy of the value of $p_L$ at which $A_N$ has a zero.

Accurate measurements of the analyzing power in the Coulomb-hadron interference region can reveal the structure of the hadron spin-flip amplitude, and give us further information on the the hadron interaction potential at large distances, as used *e.g.* in the peripheral dynamic model [20]. Our analysis shows the existence of a hadronic spin-flip amplitude, which cannot be neglected even at RHIC energies. The amplitude corresponds to the interaction of the proton with the Carbon nucleus as a whole. The ratio of the reduced spin-flip amplitude to the spin-non-flip amplitude is approximately 15%. The hadron spin-flip amplitude gives an additional positive contribution to the maximum of the CNI effect with a small energy dependence. It is

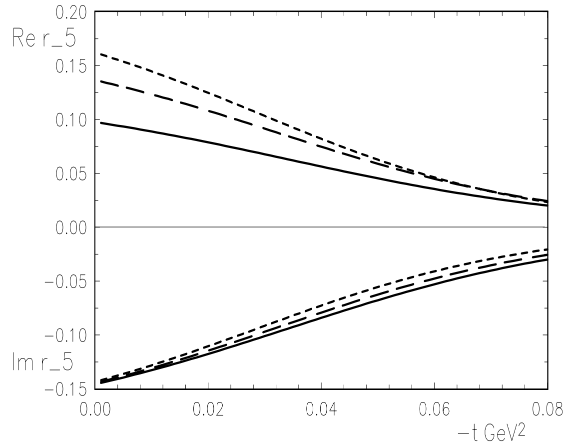


Figure 3: The $t$ dependence of $r_5$ at energies $p_L = 24$ GeV/$c$ (solid line), at $p_L = 100$ GeV/$c$ (long-dashed line) and at $p_L = 600$ GeV/$c$ (short-dashed line), without the contribution (17).

very important to carry out the experiment at higher energies, for example $p_L = 600$ GeV/$c$, and in some larger region of momentum transfer, so that we can distinguish between the various possibilities outlined in this letter.

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